NAVAL POSTGRADUATE SCHOOL

Monterey, California

COMPUTER SOFTWARE: TESTING, RELIABILITY

MODELS, AND QUALITY ASSURANCE

by

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July 1974

Technical Report

Approved for public release; distribution unlimited.

Prepared for:
Deputy Commander
rational Test and Evaluation Force, Pacific
al Air Station, North Island
Diego, CA 92135

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The work reported herein was supported primarily by Deputy Commander, Operational Test and Evaluation Force under Job Order 55730. Partial support was provided by the Navy Fleet Material Support Office under Job Order 55753.

This report was prepared by:

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE *(When Data Entered)*

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: NPS55Rh74071A
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:

4. TITLE *(and Subtitle)*: COMPUTER SOFTWARE: TESTING, RELIABILITY MODELS, AND QUALITY ASSURANCE

5. TYPE OF REPORT & PERIOD COVERED: Technical Report July 1973 - July 1974

6. PERFORMING ORG. REPORT NUMBER:

7. AUTHOR(s): F. Russell Richards

8. CONTRACT OR GRANT NUMBER(s):

9. PERFORMING ORGANIZATION NAME AND ADDRESS: Naval Postgraduate School Monterey, California

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: NC 2053 PO-3-0001 Job Order 55730

11. CONTROLLING OFFICE NAME AND ADDRESS: Deputy Commander, Operational Test and Evaluation Force, Pacific NAS North Island, San Diego, Californis

12. REPORT DATE: July 1974

13. NUMBER OF PAGES: 86

14. MONITORING AGENCY NAME & ADDRESS *(if different from Controlling Office)*:

15. SECURITY CLASS. *(of this report)*:

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:

16. DISTRIBUTION STATEMENT *(of this Report)*:

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*:

Approved for Public Release; Distribution Unlimited

18. SUPPLEMENTARY NOTES:

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*:

Software
Reliability
Testing Methodology
Quality Assurance
Reliability Growth

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*:

A study of the problems of measuring and assuring the quality of computer software is made. A quantitative measure of software quality is defined, and various mathematical models for estimating that measure of effectiveness are presented.

Software testing methodology, data requirements and data collection procedures are discussed. Also included is a discussion of the customer's role in software quality assurance.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

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SECURITY CLASSIFICATION OF THIS PAGE *(When Data Entered)*

COMPUTER SOFTWARE: TESTING, RELIABILITY
MODELS, AND QUALITY ASSURANCE.

I. INTRODUCTION

Computer systems are becoming more and more complex as faster and more versatile computer hardware evolves. The resultant sophisticated uses of the computer systems demand that programmers develop reliable instructions to drive the computer systems. Nowhere is this more evident than in the military where computers are being used increasingly as the heart of sophisticated weapons systems such as real-time command control systems which actually control their environments by receiving data, processing data and returning results fast enough to affect the functioning of their environments.

It is now the case that software costs (those costs related to developing, testing, correcting and integrating all computer programs and data descriptions used to operate, test, monitor and maintain the hardware system) now exceed the hardware costs in most complex systems. We now see huge programs with perhaps over a million words of code.

The technical literature abounds with articles about quality control procedures, test methodologies and techniques for measuring and predicting reliability--most applied to the hardware systems. Though not complete, the theory is certainly developed to such a state that highly reliable hardware can be achieved. Unfortunately, the same cannot be said about computer software. Until recently the software has received only modest attention and, as a result, software development is still more an art than a science. This is the case despite the fact that the influence of the software may well dominate the hardware when considering overall system reliability. Consider, for example, the possible consequences if a real-time military

command control system crashes because of a software deficiency while enemy units are being tracked. Even a few moments delay to restore the targets on a video output display could be vital. In this case a software deficiency, just like a hardware failure, could incapacitate a key component of our defense structure at a critical time.

The lack of attention given to software quality control has resulted, predictably, in products which are characteristically laden with software errors even after they have been released to customers for operational use. The failure of the software product to perform as required results in the loss of customer confidence for the entire system.

How can software quality be improved so that the customer's confidence in the software subsystem can be restored? We seek to provide at least a partial answer to this question in this report. As with hardware, a quantitative measure for evaluating software must be used if a meaningful assessment of software quality is to be made. More is needed than a subjective assessment of program performance and program deficiencies if quality is to be described in other than general terms such as "acceptable" or "unacceptable." This report defines a quantitative measure of software quality, and mathematical models for estimating that measure of effectiveness are presented in Chapter 4. The data requirements and data collection procedures are discussed in Chapter 3.

A numerical measure of software quality will enable us to evaluate a software subsystem, but, by itself, the measure will do nothing to improve the quality of the software delivered to the customer. To accomplish this, more effort and resources must be expended in the design, development and

integration phases of the software. The causes of "software unreliability" must be determined, and steps must be taken to alleviate the contributions of the identified causes of software problems.

Certainly, the software contractor must accept much of the blame for a poor quality software product. The contractor is responsible for the code that is created, and he must exercise management control over the product. Nevertheless, the customer can assume a more active participation in the development of the software which should help assure that his delivered software is acceptable. This can be accomplished by completely spelling out exactly what the software must be able to do and by requiring that sufficient software testing be carried out to demonstrate adequately that the software conforms to its performance specifications. In addition the customer can require strict management control of the software during its development. In Chapter 5 we discuss the role the customer can play to improve the quality of the software which is delivered to him.

Finally, we point out the need for further work in the area of software quality control to validate the mathematical models and to improve the test procedures.

II. SOFTWARE RELIABILITY

2.1 Measure of Performance

In order to provide a meaningful assessment of software quality, quantitative methods of evaluating software must be developed. Traditionally, quality assessments have been mere subjective evaluations of software based on the frequency of program deficiencies. However, subjective evaluations for software do not seem consistent with the use of the rather sophisticated methodologies used to measure the quality of interacting hardware. For complex computer systems, consisting of the hardware, software and human operator subsystems, the most widely accepted and most meaningful measure of performance is total system reliability, defined as the probability that every subsystem performs within specification limits for the time and under the conditions of intended customer use. Thus there is an obvious need to measure the reliability of the software subsystem. If no software reliability specification is explicitly stated, one must be determined from the specification for the total system. A study of the cost-benefit trade-offs would then determine the reliability apportionment.

2.2 Definitions

Although software reliability appears to be the most appropriate measure of performance, there are definitional problems because the meanings of such words as software reliability and software failures are not entirely obvious by analogy with the corresponding hardware reliability concepts. Our software reliability study will therefore begin with definitions of these basic terms.

Definition 1: A software failure occurs when an input is made or a command is given and the software subsystem does not respond as required.

It is generally obvious when a program has failed to function as required. The failure may be manifest in many ways. A complete stoppage of the system may occur; output values may fail to lie within acceptable tolerance limits of the true values; or troubles with interactive hardware, e.g., erroneous video displays or incorrect navigation, may be experienced. On the other hand, there will surely be some cases of controversy as to whether or not a failure has occurred, and some failures will go undetected. Detection of failures is, to a large extent, a subjective decision which must be made by the operators or the test personnel, hopefully on the basis of objective criteria such as performance specifications. In actual practice, failure detection depends on an operator's observation, so, in effect, a software failure is what an operator says is a failure.

After failures are detected some programmer must inspect the program and locate the causes of the failure. Logical or clerical errors in coding may be found to be guilty of producing the incorrect results. When software errors are located, action should be taken to correct the errors to prevent recurrence of the failures. Obviously, the correspondence between software errors uncovered and software failures detected is not necessarily one-to-one. Many errors may occur without a failure being detected, and a single detected failure may be a result of several software errors. Also, a software failure may be reported that is in fact no software failure at all, but rather an operator or hardware deficiency.

Because of the difficulties involved with determining a correspondence between the number of software failures and the number of errors, we choose not to define software reliability as a probability of error-free performance. Instead, we opt for definitions (depending on the type of data observed) based on the observed difficulties--failures. We offer two different definitions of software reliability depending on whether the observed data are quantitative, such as times between failures, or qualitative, such as "run success" or "run failure."

Definition 2 (Quantitative Data): Software reliability is the probability that the software subsystem will operate without a single failure for a specified length or time under given conditions.

Definition 3 (Qualitative Data): Software reliability is the probability that the software subsystem will perform without failure for an entire run under given conditions.

The latter definition requires further comment about the definition of the word "run." In some cases a run might be taken to mean the operation of the software under a particular set of input combinations. Alternatively, it may mean an operation of the software subsystem for a fixed length of time, or it may take some other meaning. Although admittedly somewhat vague at this juncture, the meaning of the word "run" should be clear in a given application from the context in which it is used.

2.3 Classification of Errors

The definition of software reliability fails to distinguish between

different classifications of failures. No doubt, software failures differ with respect to their impact on the system. The more severe failures may result in the failure of a mission, while less critical failures may only cause nuisances or limitations which have little effect on a mission's success. It may be appropriate to classify failures according to their impact on the system and to define reliability in terms of a particular class of failures, or perhaps to apply some weighting scheme to failures so that the more critical failures are weighted more heavily than are the minor failures. Although appealing, such an approach is not entirely satisfactory because of the subjective nature of the assignment of weights. In this paper we make no distinction between failures. One can still apply the models that we present to a particular category of failures by simply redefining a software failure in terms of a given failure classification.

2.4 Analogy with a Hardware Reliability Program

Because the theory of hardware reliability is developed to a relatively advanced state, it is natural to try to learn about software reliability by borrowing from the hardware reliability theory. There are certainly many similarities between the two, but a few important differences have prevented a simple direct application of the hardware techniques to software. Nevertheless, much insight can be obtained as to what kinds of things should be done with software by studying the areas of a hardware reliability program. MacWilliams [17] summarizes those areas as follows:

1. Define, observe and record failures at the system, subsystem and component levels.

2. Determine the statistical behavior of failures and develop a mathematical model for failures.
3. Isolate the principal causes of failure.
4. Determine the quantitative dependence of component failure rates.
5. Determine achievable limits of component reliability as a function of the variables which can be controlled in the development.
6. Develop a theory to combine component reliabilities into subsystem or system reliability.
7. Optimize the distribution of unreliability by considering the component reliabilities as a function of cost and by considering the abilities to compound component reliabilities into subsystem and system reliabilities. Apportion the overall system reliability among subsystems in such a way as to achieve the most economical attainment of the desired system reliability.

These same steps are desired for a software reliability program. Certainly we require a quantitative knowledge of software failure statistics, including causes and dependencies, and a mathematical model of the failures. Also, a method of compounding the reliabilities of software modules into a "total software reliability" would be desirable, as would a theory for apportioning effort among software modules. Unfortunately, difficulties result when one tries to apply all steps of a hardware reliability program to software. Even when one attempts to apply the first few steps, basic differences between hardware and software failures create a need for new mathematical

models. Let us examine those differences.

2.5 The Random Nature of Software Failures

Hardware failures occur randomly with time as the hardware deteriorates. However, there is no degradation of software, and there is no physical mechanism which generates software failures. Once all errors are removed, the software is 100 per cent reliable and will remain so forever, provided no program changes are made. What then accounts for the randomness of software failures?

Different input combinations result in different requirements of the software. The paths traversed within a software program depend on the particular input combinations, and each path can be thought of as containing possible software bugs waiting to be discovered. Without correction, the same errors will occur each time a specific logic path is traversed. If the errors result in an observable software failure, the given failure can be reproduced at will, or it can be avoided by operator control of the input combinations. Therefore, software failures are functions of the input combinations--not random functions of time. However, in practice, input combinations are chosen in a somewhat random fashion, and the resultant effect is that errors are uncovered and failures are observed at random. It is in this sense that we talk about the random occurrence of software failures. Thus, although there are conceptual differences between software failures and hardware failures, software failures are surely the analog of hardware failures that we should use to measure software reliability.

2.6 Restrictions on the Mathematical Reliability Models

In Chapter 4 we present several mathematical models for software reliability--all based on software failure data. Because of the absence of degradation of software and the "find and fix" actions that are taken when errors are discovered, the models assume a reliability growth as a function of total test time. The models consider the reliability of the software subsystem alone. No attempt is made to model the interactions of hardware-software, operator-software or hardware-software-operator. These are all important considerations, but software reliability being in the embryonic stage that it is requires us to focus singly on it at this time. This does not preclude the combination of software reliability with hardware and operator reliabilities to obtain an estimate of overall system reliability.

Our models treat the software subsystem pretty much as a black box in the sense that the internal structure of the software is completely ignored. Again, better reliability models could probably be developed if that structure were considered. This is a refinement that would probably follow once a good understanding of software reliability is obtained and expertise is developed.

III. SOFTWARE TESTING AND DATA COLLECTIONS

3.1 Introduction

A numerical estimate of software quality can be no better than the data from which it is determined. To estimate software reliability, detailed information about the frequency of failures, the times that failures occur and the severity of the failures is needed. Furthermore, if there is to be reliability growth, the causes of the failures must be identified, and corrective action must be taken. Only through good, representative failure data can reasonably accurate mathematical models of software reliability be developed and reliable predictions about software quality be made.

The software test effort has already become the single most costly step in most software production processes. The high cost of testing combined with the high reliability requirements of complex systems demands that efficient test methodologies be developed. It also requires that the data reporting system be established early in a software test program so as not to lose valuable information.

3.2 Stages of Software Testing

The development of software is a "bottom-up" procedure. First, modules are coded. These are then combined to form functional groups (processes) which, in turn, are integrated into the software subsystem. The "total system" is then formed by integrating the software, hardware and operator subsystems. This development procedure is depicted in Figure 1.

STAGES OF SOFTWARE DEVELOPMENT

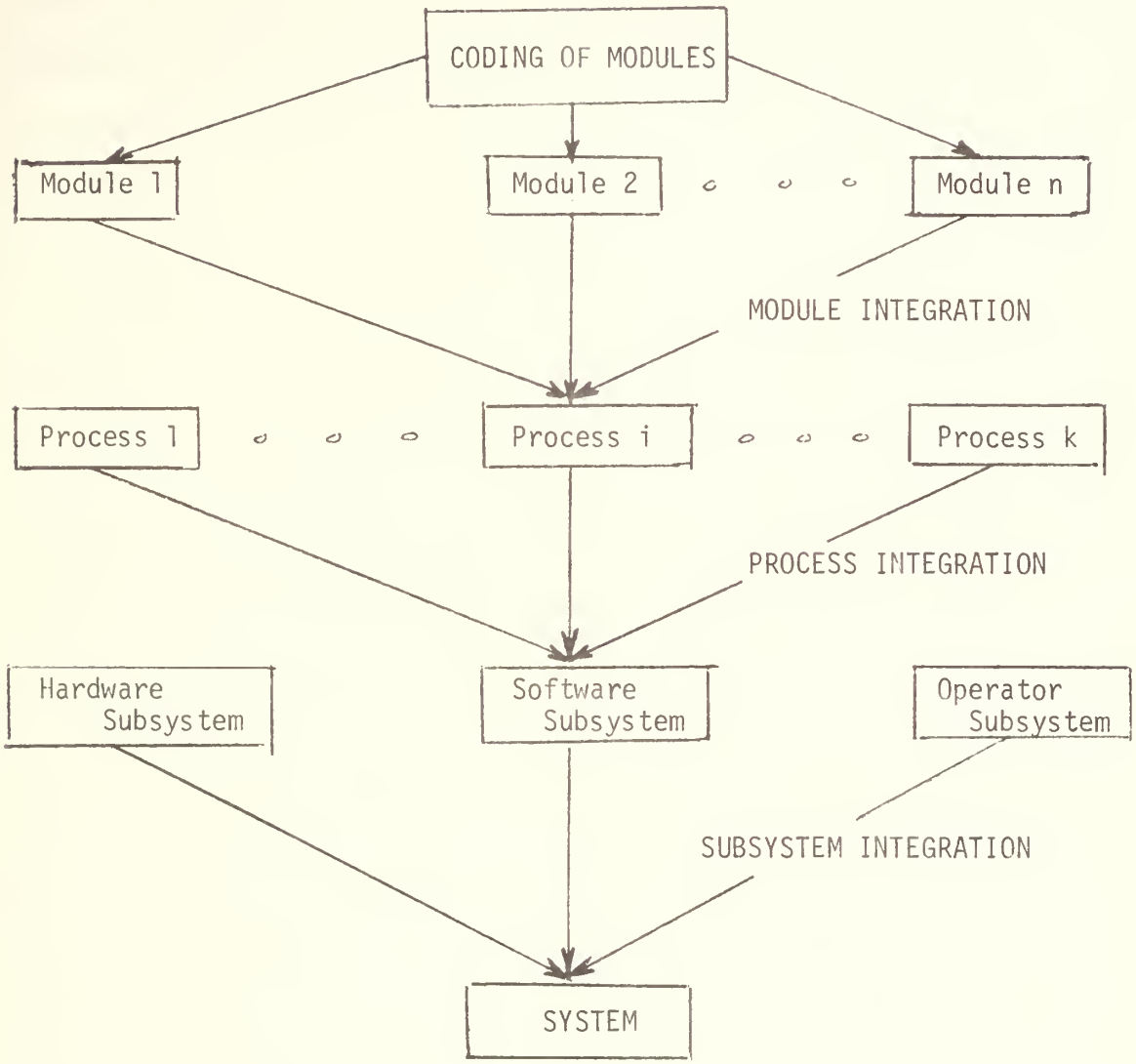


Figure 1

Software testing should be conducted throughout each stage of development. At the lowest level, the primary purpose of testing is to discover the crude errors such as compilation errors and syntactical errors. Debugging at this level is certainly much simpler than at the process or subsystem levels. Because modules are usually of manageable size, having perhaps only a few functions, module check-out which is nearly exhaustive would be possible in many cases. The module failure data are useful for determining statistical estimates of quality which could be employed to establish additional test requirements for the module.

When several modules are integrated to form a process, or when several processes are integrated to form the software subsystem, problems surface which had previously gone undetected. This occurs because the complex interactions between modules could not have been tested previously as the individual modules were separately checked out. Unfortunately, the complexity of the module interactions precludes the direct use of module reliability estimates to determine the software subsystem reliability, or even the process reliabilities. The multitude of branching possibilities in a typical process or software subsystem and the complex interactions seem, at this time, to make infeasible a computation of subsystem reliability from a knowledge of module reliabilities as is done with hardware systems in a "series/parallel" sort of analysis. For these reasons, tests must be run, and failure data must be collected during and after the integration stages.

At the beginning of the integration stage many bugs will likely be experienced and frequent failures will occur. As with the module failure data, this information is what is needed to estimate present reliability and

to provide guidance as to the extent of testing which must be performed.

Later in the integration stage, after the program has been debugged to a point that it will run for some time before failing, the software should be subjected to some sort of simulation test by a functional test program to make sure that a software subsystem which may perform without failure does indeed do the job for which it was designed. This provides an opportunity to test the software in a complete system with hardware and operator interfaces. It is an ideal time to collect failure data and to measure the reliability of the software subsystem. At this stage, proper design of test plans and rational methods for validating programs become critical. Anything like exhaustive testing is virtually impossible because the number of possible cases may total in excess of several million.

Finally, after the software subsystem has been judged to be acceptable and all performance requirements have been demonstrated, it is turned over to the user for field tests. If usual experience prevails, the supposedly good software now suffers a completely new set of failures induced by the unique characteristics of actual operation not considered in previous tests. The closer the test environment simulates the operational environment, the more accurate will be the reliability estimate made at the end of the integration stage.

It is the reliability estimate of the system in the operational environment that is of interest to the user. An estimate of this reliability is needed to determine if the software is of sufficient quality to allow user access. If the field tests are unsatisfactory, the software may have to be returned to the developer for corrective work and more testing. The

release of an unreliable product will result in the loss of users' confidence.

3.3 Test Methodology

We have discussed the importance of the test effort, and we have mentioned that software testing is becoming the single most costly element in the development of software for complex systems. Because of the importance and expense of software testing, efficient test procedures must be used. Despite the need, no general systematic test methodology is available which can be applied to test each program to determine whether or not all software components perform as required.

There have been some recent attempts (see, for example, London [16] and King [12]) to develop procedures for actually proving the correctness of programs. Presently, these procedures are only applicable to relatively small programs written in special languages. There seems to be little hope that generally applicable methods for formally proving programs logically correct can be developed for large complex programs.

With the large number of input combinations that need to be examined, it would be desirable to have a computer program that could be used to check out a software subsystem by exercising all options and all branches within the subsystem through all the feasible ranges of values. The tester would only have to supply parameters to such a program and the automatic computer test program would do the rest of the work. Jelinski and Moranda [10] point out that this is not possible because even with our latest generation of computers with a nanosecond cycle the number of possible input combinations involved in an average-size software program exceeds 10^{23}, and, consequently,

the processing time exceeds the astronomical limit of 10^{14} seconds of computer time. Thus, a completely automatic computer test program does not appear to offer a feasible solution to the software testing problem. Certainly, software checkout could be improved if assistance could be provided by some sort of computer program tester. Jelinski and Moranda [10] report of such a tool, called a "program testing translator", under development at McDonnell Douglas Company. The translator, currently designed to run only with FORTRAN programs, when exercised with a software program will count the number of times each branch in the program was executed by a given set of input conditions. It also performs a number of counts on various types of statements. These counts provide a good indication of which branches have been checked out for a given range of values, thus providing assistance toward achieving reliability. Such a "program testing translator" does not completely solve the test problem, even for FORTRAN programs, but it is certainly a useful tool for testing software.

The question of how a set of input conditions should be selected to test a software program remains to be answered. The answer depends on many factors such as:

(1) the size of the program being tested,

(2) the number of tests that can be run,

(3) the frequency of use of the various functions comprising the program, and

(4) the criticality of the functions to mission success.

Because failures do not occur randomly with time, but rather they occur because of the traversals of different paths through the program, the tests must include enough cases to exercise as many paths as possible consistent

with the resources available for testing and the reliability objectives. No doubt, many of the software errors would probably be detected if input combinations were selected at random. In fact, such a procedure is often used in the early stages of a test program to detect the crude errors which account for a large percentage of all of the errors. However, if the tests are to demonstrate that all functions perform as required and that each performance specification is satisfied, random selection of tests is not satisfactory. Furthermore, if the test data are to be useful for estimating software reliability, the test cases must consider the criticality of the various functions and the frequency of occurrence of the functions during program operation. Otherwise, the testing would not be representative of actual operation. This would result in a bias in the estimation of software reliability.

In summary, the input combination sequences selected for testing software should be determined by analysis of the performance criteria, the frequency of use and the impact of the functions on mission success. The tests should be conducted in an environment which simulates as closely as is economically feasible the conditions that would be experienced in actual operational use. Efficient testing requires that the tester be knowledgeable about the use of the system and be cognizant of all performance specifications, both implicitly and explicitly stated. Keezar [11] suggests that a sufficient, though cost-effective number and variety of input messages must be examined in order to exercise the critical system limits, interface areas, timing factors and storage allocations. Also, a number of likely occurrences of illegal system inputs should be used in an attempt to make the system fail.

Finally, the tests must be strictly controlled, reproduceable and documented in depth.

3.4 Data Collection

In order that maximum information be acquired from each test run, good detailed data must be collected. Although everyone expresses interest in software reliability, very few people seem interested in documenting software failures. At least it has been historically true that very little software failure data have been collected that are useful for an analysis of software reliability. This want for useful data is partially responsible for the poor quality of delivered software. It has also handicapped the theoretical development of mathematical models of software reliability. What software models that exist have been developed primarily on the basis of what appear to be plausible assumptions about failures or errors. The real test--the scrutiny of a model in light of actual data--is yet to be made in most cases. All too often, the reliability analyst has been asked to work in a virtual vacuum without any usable data.

In addition to the reliability analysis, data are required for the detection and correction of errors. Certainly, one objective of running software tests is to uncover bugs so that the reliability of the software will grow as cumulative test time increases. Without complete documentation of failures this reliability growth may not take place. The data also provide a measure of the extent to which the software performs as required, and it provides us with a measure of the amount of additional testing that is required.

What data should be collected? If the data are to be useful for all of the above purposes--reliability estimation, error detection and correction, and software validation--the necessary information includes the following:

(1) a description of the test run (including the input data),

(2) the date and time of the run start,

(3) the date and time of the failure incident,

(4) the date and time of the system restart,

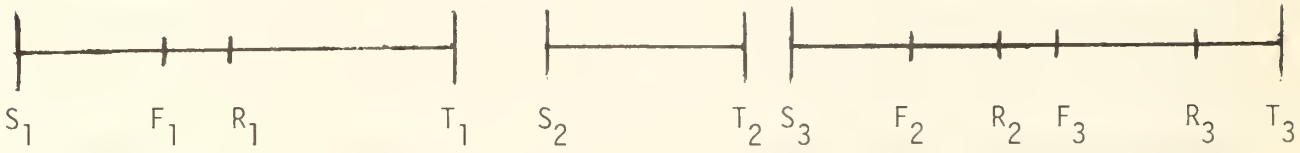
(5) the date and time of the normal termination of a run,

(6) the impact of the failure on system performance,

(7) the traffic load and possible environmental influences, and

(8) a detailed description of the problem.

For the single purpose of a reliability analysis, where we are concerned with the times between failures, we are mainly interested in the time trace of starts, failure occurrences, restarts and normal run terminations. All that is needed in addition is a determination of the type of failure--whether it be a software deficiency, a hardware malfunction, or an operator error. An example of the sort of time trace that is desirable is depicted in Figure 2.



where S_i = time run i started

T_i = time that run i terminated

F_i = time of the i^{th} failure

R_i = time of the i^{th} restart

FIGURE 2

The following information can be constructed from a time trace:

(1) the distribution of the time between failures,

(2) the mean time between failures,

(3) the probability of operation for a given interval of time without failure,

(4) the mean time to restore,

(5) the probability of a successful run, and

(6) software availability.

Since we define software reliability as the probability of a failure-free run for a given time interval when data are times between failures, this information is exactly what we are interested in.

In some cases we may wish to ignore some of the characteristics of the data and use it solely to classify a given test as a success or a failure. This may be necessary if it is too costly to install automatic recording equipment, or if it is impractical to have an observer make the continuous observations required to provide a time trace. We then must be satisfied with simply counting the number of failures at the ends of discrete periods of time or to count the number of failures in a given number of test runs.

The remainder of the information that we record is that needed to validate the software and to classify, detect and correct errors. When collecting data, we must keep in mind that any information that would enable a programmer to recreate the problem, to locate the cause of the problem or to correct the problem should be provided. This requires a complete description of the test run, the manner in which the problem was manifest and the impact of the problem on system performance. A statement about the recovery or bypass procedures

and a computer core dump would also be useful for the subsequent examination and corrective action that must take place. The failure report should be followed by a supplementary report of the remedial action that was taken to patch the program. The documentation associated with a given failure should be considered complete only after the remedial action report has been issued.

We have implied that failure data are useful for debugging the software, for validating the software and for estimating software reliability. We concentrate on the latter item in the next chapter. Several mathematical models for software reliability are presented, some of which require quantitative data and some which require only qualitative data.

IV. MATHEMATICAL MODELS

4.1 Introduction

Quantitative criteria are needed to assess the quality of software. We have remarked that the software failure data provide the most important indication about the quality of the software. It is rarely, if ever, the case that any large complex software, subsystem is completely debugged. Therefore, the strongest statement that can usually be made is a statement about the probability of a failure-free operation - a reliability statement. In this chapter we present several mathematical models, each of which attempts to provide an estimate of the reliability of the software sybsystem.

In most cases reliability specifications for the total system are established in advance of software development. Sometimes the software subsystem reliability specification is stated explicitly; other times it must be determined from the overall system requirement. In all cases, it is handy to have some reliability specification against which to measure progress and to determine test requirements. We must have some realistic goal to shoot for so that we can judge when the software is of sufficient quality to allow user access. The difference between the attained reliability and the reliability objective should be the feedback information which is used to determine the extent of the testing which must follow.

Because of the magnitude and complexity of some software subsystems it may be desirable to apportion the software reliability specification into module or process reliability requirements. There is a relatively well developed theory for the reliability apportionment with regard to hardware, but, as yet, little work in this area has been done with software. We do not address the problem of reliability apportionment, nor do

we comment about what the reliability specification should be. These are important areas which depend primarily on the user's needs, the criticality of the system and economic considerations. We will assume that explicit provisions for software reliability have been established.

In the work that follows we assume that the software development is at a point where configuration control has been instituted so that all subsequent changes incorporated into the software must be formally approved and documented. This should occur by the time that the processes are integrated to form the software subsystem. During earlier stages the failure behavior of software will probably be so erratic that no meaningful mathematical model could be developed. For the purpose of predicting operational reliability, the appropriate stage of testing over which failure data should be collected is the period of actual system operation. However, reliability estimation is needed prior to that stage so that progress can be measured and test criteria established.

We make no distinction between software failures as to criticality. Certainly, a failure which yields the system inoperable is more important than one which merely causes a nuisance. Nevertheless, classification of software failures according to severity requires, to a large degree, a subjective assessment. We prefer to restrict our attention to the problem of estimating reliability where all failures are weighted equally. If failures were classified as to severity, nothing would prevent one from using the models that we present for the case where a software failure has been redefined to include only those failures of a given criticality or worse.

The primary data that we require is the time trace of software operation discussed in the previous chapter. In some cases the actual times between failures will not be necessary. Instead, qualitative data such as "success" or "failure" may suffice.

We assume that the reader has some familiarity with the terminology used in reliability analysis. Consequently, many standard reliability terms will be used without definition or elaboration. A review of the basic theory of reliability is presented in Appendix A.

4.2 A Hardware-Oriented Approach

We consider first an approach towards developing procedures for estimating software reliability which borrows heavily from the techniques of hardware reliability analysis. It is natural that we should try to exploit the vast reservoir of reliability theory already well established and validated for hardware systems (especially since we have observed analogies between software and hardware failures). When applied to software, many of the ideas of hardware reliability theory carry over without change. There are, however, some differences which require that some caution be taken. For example, we have already discussed the differences in the nature in which we speak of the random occurrence of failures. Furthermore, unlike hardware, there is no degradation of software due to age. If all bugs were removed from the software, it would have a reliability of one thereafter. Finally, because of the debugging that takes place as failures occur and errors are detected, there is a natural reliability growth that accompanies testing and operation. This reliability growth results in changes in the distribution of times between failure and consequently changes in the

reliability function itself as cumulative test time increases. Later in this section, we comment about steps that should be taken to accomodate those changes.

The procedure that we describe follows that suggested by SCHNEIDEWIND in [24] and [25], where he applied a hardware reliability approach in an analysis of the Naval Tactical Data System (NTDS) software. We outline his procedure as follows:

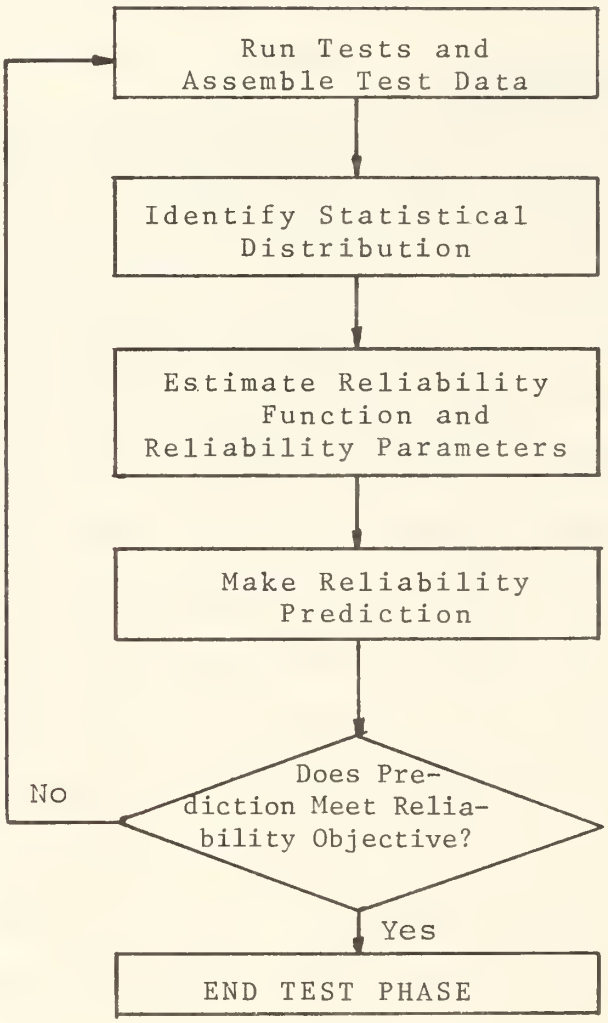


Figure 3

As indicated by Figure 3, the first step is to run tests and assemble the failure data. The failures must be classified as to cause; whether they be operator, hardware, software or unknown. In addition, if there is to be a distinction as to severity, the software troubles are to be divided into groups according to criticality. This step yields a set (or several sets) of times at which software failures were observed, say $A = \{t_1, t_2, \ldots, t_n\}$ (or, $A_i = \{t_{i1}, t_{i2}, \ldots, t_{in_i}\}$ for each level of severity i) .

For the identification phase of the procedure, the reliability analyst relies on theoretical principles, physical considerations, and previous experiments to rationalize the nature of software failures. Furthermore, the analyst should arm himself with plots of various empirical functions to provide clues as to the type of probability functions that might be appropriate. The shapes of the relative frequency function for the times between failures, the empirical reliability function and the empirical failure rate function, combined with the theoretical considerations and studies of failures of other software, should suggest an hypothesis about the theoretical reliability function. That hypothesis must then withstand further statistical examination. (For a good discussion of the empirical functions, the reader is directed to GNEDENKO [9, pp. 78-95].)

Once the analyst has formulated an hypothesis about the reliability function, or, equivalantly, the probability density of time between failures or the failure rate function, he must then obtain estimates for the parameters of the appropriate functions. For example, if the exponential law, $R(t) = \exp(-\alpha t)$, is suggested, the analyst must obtain an estimate of the single parameter α . In other cases like the Weibull law, he may have to

estimate two or more parameters. He is thus led to consider the basic problems of mathematical statistics: (1) the estimation of the values of unknown distribution parameters and (2) the verification of statistical hypotheses. The statistical techniques required to solve these problems can be found in most books on statistical inference or in reliability books such as LLOYD and LIPOW [15] or GNEDENKO [9].

Once the parameters have been estimated, a test of the hypothesis is performed. The Kolmogorov-Smirnov (K-S) test is one of the simpler and more powerful tests that can be employed to measure the goodness of fit of the assumed theoretical distribution to the empirical distribution. If the hypothesis is not rejected by the statistical test, the analyst then proceeds to estimate the reliability function and to make a prediction of the software reliability. On the other hand, if the hypothesis is rejected, another is formulated, and the procedure is repeated.

The "acceptance" of the hypothesis completely determines the reliability function, for the reliability parameter(s) will have already been obtained. Because statistical estimates of the unknown parameter(s) are being used, the reliability analyst should provide a confidence interval for the value(s) of the parameter(s) of the reliability function. To be conservative, the lower confidence limit on the reliability function should be utilized in the reliability predictions. On comparing the predicted reliability with the specified or desired reliability, the analyst determines whether additional testing and debugging is required. The magnitude of the difference between predicted and specified values should serve as an indicator about the extent of additional testing (see, for example, SCHNEIDEWIND [26]).

If debugging and additional testing are undertaken the entire procedure is repeated. Some difficult statistical problems arise at this time because of the natural reliability growth that accompanies the testing as errors are detected and corrected. The effect of the reliability growth may be a simple change in the value(s) of the reliability parameter(s). On the other hand, the distributional form of the reliability function may change. For example, let T be the random variable denoting the time to the next failure, and let $R_1(t)$ be the reliability function after a comulative test time of length S_1. Then, let $R_2(t)$ be the reliability function after a cumulative test time of length $S_2 > S_1$. The first case, where only the reliability parameters change is illustrated by Figure 4; the second case where the form of the distribution changes is depicted in Figure 5.

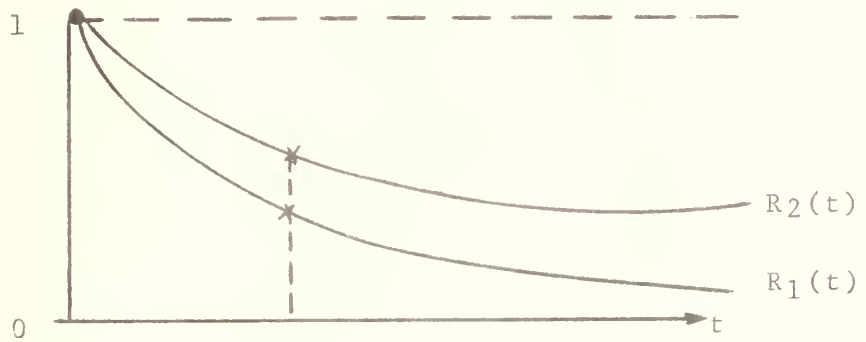


Figure 4

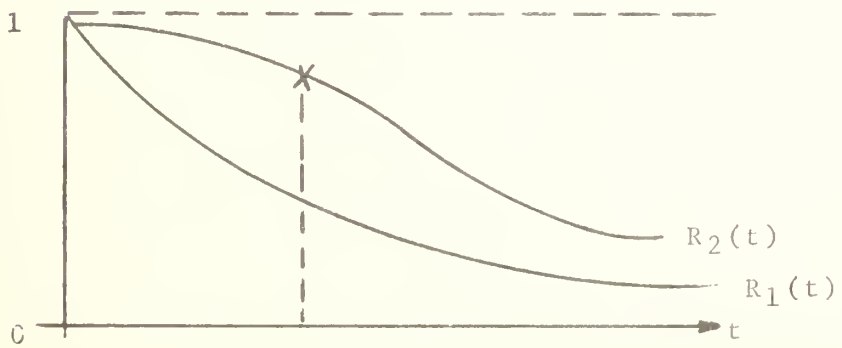


Figure 5

In both figures we observe that the reliability has grown, i.e. $R_2(t) > R_1(t)$ for all t. This is certainly the objective of additional testing, but the growth does make the problem of estimating reliability more difficult. The difficulty arises when the analyst tries to determine what data is to be used as a basis for the estimation of the "new" reliability function. Because of the changes with respect to cumulative test time, some of the data set will probably not be representative of the current state of the process generating failures. We need to test to see if significant changes have taken place. Suppose, for example, that we have two sets of observed times to failure, say $A_1 = \{t_{11}, t_{12},\ldots,t_{1m}\}$ and $A_2 = \{t_{21},\ldots,t_{2n}\}$ where A_1 and A_2 were collected during test periods one and two as shown in Figure 6.

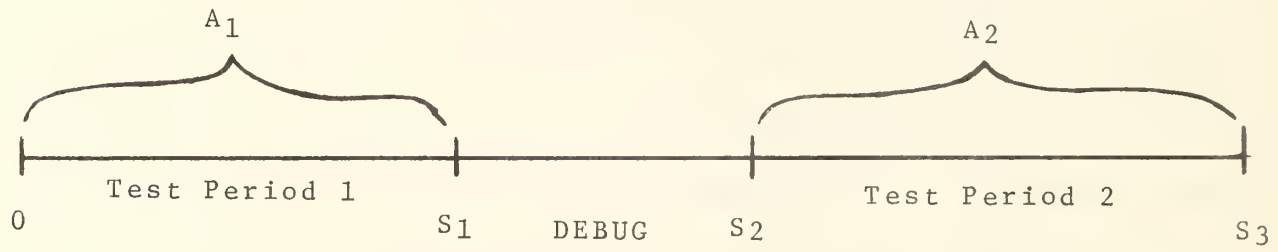


Figure 6

The observations of set A_1 were made on a random variable with probability distribution $F_1(t)$, and the observations of A_2 were made on a random variable with probability distribution $F_2(t)$. Hopefully, $F_1(t) \geq F_2(t)$, so that the debug period has actually improved the software. There are

several easy-to-apply statistical tests that can be used to test the hypothesis that no improvement has taken place; i.e.,

$$H_o : F_1(t) = F_2(t)$$

against the alternative

$$H_a: F_1(t) \geq F_2(t)$$

The nonparametric tests such as the sign tests, Wilcoxon's test and Smirnov's test are all useful for testing the given hypothesis.

If the statistical tests fail to reject the hypothesis that $F_1(t) = F_2(t)$, all the data can be pooled together to obtain our revised estimate of reliability. However, if there are indications that a change has indeed taken place, we certainly want to weight the latest data more heavily. If the earlier data are ignored entirely, the result is a reduced sample size. On the other hand, if nonrepresentative data are used the resultant reliability predictor may not be accurate. Research is needed in the area of developing smoothing techniques for weighting the different sets of failure data.

When the reliability function is revised and the reliability prediction is updated, the prediction is again compared with the desired reliability. This procedure continues until the predicted reliability reaches the specified value.

Based on its application to NTDS data, the "hardware-oriented" reliability approach described in this section appears feasible. (see SCHNEIDEWIND [24].)

However, the methodology requires validation against other sets of data before any general conclusions about the approach can be made.

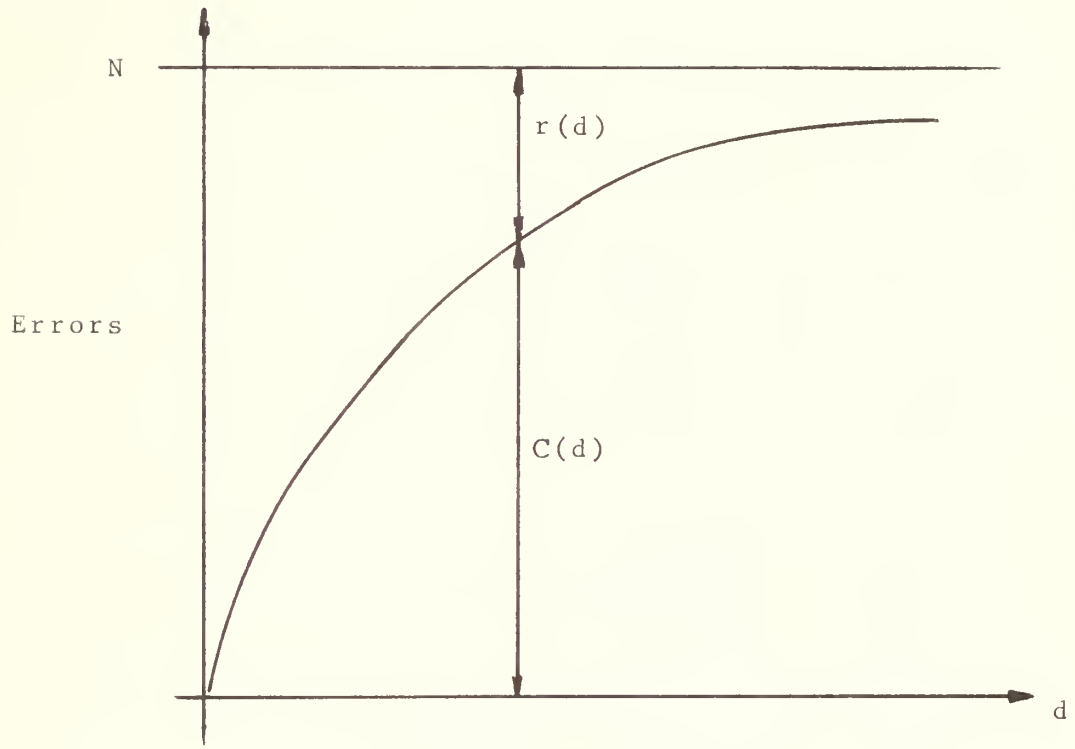
4.3 Error-Counting Models

In the preceding section we pointed out some difficulties which result from the possible non-homogeneous (time variant) nature of the data collected over different test periods as software errors are detected and removed. What is needed is a mathematical model for software reliability which is tailored to the special characteristics of software failures and which explicitly accounts for the natural growth of software reliability as a function of cumulative test time. The models presented in this section attempt to fill this need.

Assume that the total number of errors in the software program at the start of the test period (preferably the integration test period) decreases directly as errors are corrected. If the cumulative number of errors corrected during debugging is recorded, then the number of remaining errors is simply the difference between the initial number of errors and the number corrected. This assumes that no new errors are introduced during debugging. Let N be the (unknown) initial error count, d the cumulative debugging time since the start of the test, $C(d)$ the total number of errors corrected in $(0,d)$ and $r(d)$ the total number of errors remaining in the software after a cumulative debugging time of length d. Then, it is clear that

$$r(d) = N - C(d) \tag{4.3.1}$$

This is illustrated by Figure 7.



ERROR MODEL

Figure 7

The reliability model that we present is basically that developed by both SHOOMAN [27], [28] and JELENSKI and MORANDA [10]. Their models assume that the probability of an error being encountered in a small interval of time of length Δt after t hours of successful operation (the failure rate $Z_d(t)$) is proportional to the number of remaining errors. Mathematically,

$$Z_d(t) = K \cdot r(d) \tag{4.3.2}$$

for some constant of proportionality K. The proportionality constant may vary from program to program depending perhaps on such factors as the total number of machine language instructions, the rate of processing instructions, the software structure and/or the type of test procedure.

We can then write the reliability function (see Appendix) as

$$R(t) = \exp\left(- \int_o^t Z_d(X)dX \right) \tag{4.3.3}$$

On substituting (4.3.1) and (4.3.2) into (4.3.3) and writing the reliability function as $R(t,d)$ to indicate its dependence on both t and the debug time d, we get

$$R(t,d) = \exp(-K(N - C(d)t) \tag{4.3.4}$$

We note that (4.3.4) is the exponential reliability model with reliability parameter $\alpha(d) = K(N - C(d))$. Consequently, the mean time between failures (as a function of d) is

$$MTBF(d) = 1/\alpha(d) = 1/K(N - C(d)) \qquad (4.3.5)$$

Since $C(d)$ is assumed known, we must determine only the constants K and N.

Using the approach of SHOOMAN [28], suppose that out of n total test runs there are s successful runs and $n - s$ unsuccessful runs. Let $T_1, T_2, \ldots, T_s$ represent the hours of success for the s successful runs and $t_1, t_2, \ldots, t_{n-s}$ the run hours before failure for the $n - s$ unsuccessful runs. Then the cumulative run hours is

$$H = \sum_{i=1}^{s} T_i + \sum_{i=1}^{n-s} t_i$$

An estimate of the mean time between failures is then obtained from the ratio of total run hours to the number of failures

$$MTBF \tilde{=} 1/\hat{\alpha} = H/(n-s) \qquad (4.3.6)$$

The unknown constants N and K can be evaluated by looking at the estimate (4.3.6) after two different debugging times $d_1 < d_2$ chosen so that $C(d_1) < C(d_2)$. Using the method of moments, we equate (4.3.6) for debug times d_1 and d_2 to get:

$$H_1/X_1 = 1/\hat{\alpha}_1 = 1/K(N - C(d_1)) \tag{4.3.7}$$

and

$$H_2/X_2 = 1/\hat{\alpha}_2 = 1/K(N - C(d_2)) \tag{4.3.8}$$

where X_1 and X_2 are the number of software failures detected in H_1 and H_2 hours, respectively, of total run time. The ratio of (4.3.7) to (4.3.8) gives an estimate of N:

$$\hat{N} = (\hat{\alpha}_2 C(d_1) - \hat{\alpha}_1 C(d_2))/(\hat{\alpha}_2 - \hat{\alpha}_1) \tag{4.3.9}$$

We can then obtain an estimate of K by substituting (4.3.9) into (4.3.8). This gives

$$\hat{K} = \hat{\alpha}_2/(\hat{N} - C(d_2)) \tag{4.3.10}$$

The "hats" above the parameters α, N and K indicate that they are estimates of the parameters. The estimates (4.3.9) and (4.3.10) are simple functions of the cumulative errors corrected and the sample means $1/\hat{\alpha}_1$ and $1/\hat{\alpha}_2$. However, the statistical properties of estimates obtained

by the method of moments, as were (4.3.9) and (4.3.10), are not as good as those obtained by the maximum likelihood technique. Solving for the maximum likelihood estimates (MLE's), we obtain for two tests with n_1 and n_2 runs and H_1 and H_2 total hours,

$$\hat{N} = \frac{(n_1 + n_2)/\hat{K} + (C(d_1)H_1 + C(d_2)H_2)}{H_1 + H_2} \tag{4.3.11}$$

and

$$\hat{K} = \frac{n_1/(\hat{N} - C(d_1)) + n_2/(\hat{N} - C(d_2))}{H_1 + H_2} \tag{4.3.12}$$

Numerical methods are required to solve these equations for $\hat{N}$ and $\hat{K}$, but the results obtained should be superior to those given by (4.3.9) and (4.3.10). A simple iterative procedure should suffice to solve (4.3.11) and (4.3.12). First, use (4.3.10) as an initial estimate of $\hat{K}$ and substitute this value into (4.3.11) to get an estimate of $\hat{N}$. Then substitute this value for $\hat{N}$ into (4.3.12) to get a new estimate of $\hat{K}$. Repeat this iterative procedure until successive estimates of $\hat{N}$ and $\hat{K}$ do not change. (If $\hat{N}$ is rounded to an integer the convergence should be quite rapid.)

Large sample estimates of the variance of the MLE's have been determined by SHOOMAN [28]. They are given by:

$$\text{Var } \hat{K} = \hat{K}^2/(n_1 + n_2)$$

and

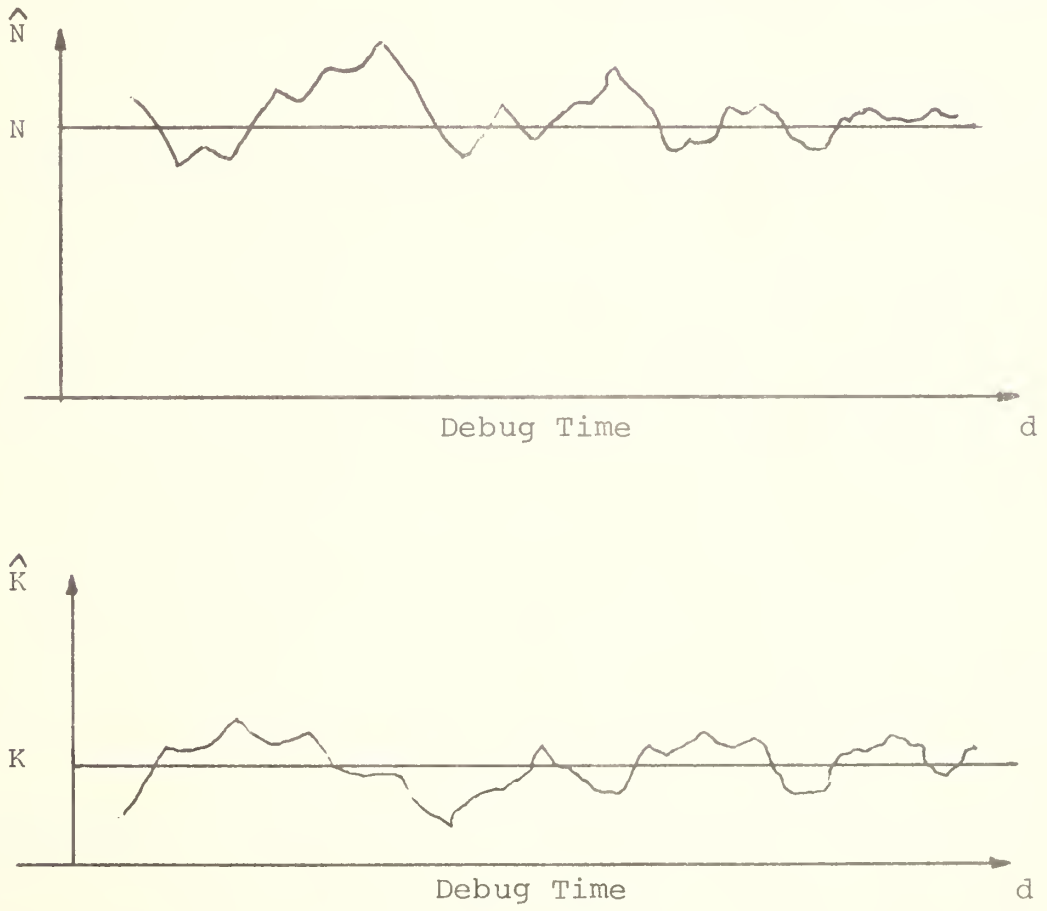
$$\text{Var } \hat{N} = \frac{1}{(n_1/r^2(d_1)) + (n_2/r^2(d_2))}$$

Other reliability models where the failure rate is taken as a function of the number of errors remaining in the software have been proposed. That of JELENSKI and MORANDA is basically the same as the above model. They do obtain different estimators for the constants $\hat{K}$ and $\hat{N}$ however. SCHICK and WOLVERTON [23] assume a model which takes the failure rate to be proportional to the number of remaining errors and which increases with operating time t,

$$Z_d(t) = A \cdot r(d) \cdot t$$

where A is a constant of proportionality. This model, which has, for each fixed d, an increasing failure rate, leads to the Weibull reliability law. If one views the test operation as a series of different runs which gradually closes in on the remaining errors, such an assumption of an increasing failure rate is reasonable. However, under normal operation, runs are not selected to examine exhaustively all possible paths through the entire range of values for all inputs. Instead, failures are caused when a particular combination of input data and path is experienced. Thus, an argument could also be made that the failure rate is constant and the times between failures have no "memory". In order to select among the models, tests must be conducted using actual failure data. One easy way to test the model

(4.3.2) is to run m operational tests and compute estimates of N and K for adjacent values d_i and d_{i+1}. Then, if the constant-failure-rate hypothesis is true, graphs of $\hat{N}$ vs d and $\hat{K}$ vs d should appear as random fluctuations about the horizontal lines through N and K, respectively, as illustrated in Figure 8. Any deviation from the horizontal pattern would suggest that the hypothesis is false. If there is no evidence to contradict the hypothesis, the m sets of data can all be pooled to obtain the estimates $\hat{N}$ and $\hat{K}$ through obvious modifications of (4.3.11) and (4.3.12).



FLUCTUATIONS OF $\hat{N}$ AND $\hat{K}$ WITH DEBUG TIME

Figure 8

Implicit in Model (4.3.4) is the assumption that all time periods of equal length represent equal intensities of testing and debugging. In reality, this is rarely the case because of varying manpower assignments and different types of testing. JELENSKI and MORANDA [10] and SHOOMAN [27] offer refinements to the basic model to adjust for unequal intensities of testing and debugging. Their refinements require basically that the previous results be normalized with respect to manpower and that the time to failure observations be normalized to account for variable exposure rates. The refinements may improve on the basic model, but, for the most part, the additional data required are just not available. Consequently, their implementation would require a great deal of subjectivity by some decision maker.

4.4 An Error-Seeding Model

The preceding reliability models rely strongly on the estimation of the number of errors remaining in the computer program after various stages of the testing process. MILLS [19] suggests a rather novel approach for estimating that quantity. He proposes that software errors be intentionally introduced at random into a program. The "seeded" errors would then be used to calibrate the testing process and to estimate the number of remaining "indigenous" errors. Although it may seem a paradox to introduce errors in an effort to remove eventually all indigenous errors, such a procedure does have a firm statistical basis.

Suppose that the software contains n_i indigenous errors, and n_s errors (seeded errors) are deliberately inserted randomly into the software. Suppose, further, that a testing process to find and remove errors is undertaken and that each remaining error - indigenous or seeded - is equally likely to

be discovered at any point of the testing process. Then, after removing a total of r errors, the probability that s are seeded errors and $r - s$ are indigenous errors is given by

$$q_s(n_i + n_s) = \frac{\binom{n_s}{s}\binom{n_i}{r-s}}{\binom{n_i + n_s}{r}} \qquad (4.4.1)$$

for $s \leq n_s$ and $r \leq n_i + n_s$. The problem is that n_i is unknown.

Let us now see how the probabilities (4.4.1) can be used to give a simple estimate of n_i. First, intuitively, it seems logical that the ratio of $r - s$ to s should be approximately the same as the ratio of n_i to n_s because of the assumption that errors are equally likely to be discovered. That is,

$$\frac{r-s}{s} \cong \frac{n_i}{n_s}$$

or

$$n_i \cong \frac{r-s}{s} \cdot n_s \qquad (4.4.2)$$

FELLER [6] provides statistical support for the estimate (4.4.2). He shows that the maximum likelihood estimate of n_i is the integer part of (4.4.2). That is,

$$\hat{n}_i = \left[\frac{r-s}{s} \cdot n_s\right] \qquad (4.4.3)$$

Example: Suppose that 100 errors are inserted into the software and, in the ensuing testing, 15 errors, consisting of 10 seeded errors and 5 indigenous errors, are found. Then $n_s = 100$, $s = 10$ and $r = 15$. The maximum likelihood estimate of n_i is

$$\hat{n}_i = [\frac{15-10}{10} \cdot 100] = 50 .$$

Being only a statistical estimate, the actual number of indigenous errors may be more or less than 50. We can test the hypothesis H_0: $n_i = 50$ against the alternative that $n_i > 50$ by using the probability function (4.4.1). We would want to reject H_0 only if the number of seeded errors among the 15 discovered errors were "too small." For example, suppose the 15 discovered errors included only five seeded errors. Then, assuming H_0 is true, the probability of obtaining five or fewer seeded errors is given by,

$$p = q_0(150) + q_1(150) + \ldots + q_5(150) .$$

The probability p is difficult to calculate exactly, but, using the binomial approximation, we find that p is approximately 0.01. With such a small probability, we would be inclined to believe that the number of indigenous errors is larger than 50.

In the example, we see that we can obtain the MLE for the number of indigenous errors, and we can also use the probability function (4.4.1) to test hypotheses about the magnitude of n_i . The test of hypothesis is complicated, however, by the mathematical difficulties experienced when working with (4.4.1) . The binomial or normal approximations are only good when r is small compared to $n_i + n_s$, but in cases of practical interest we would like that r be nearly as large as $n_i + n_s$. Consequently, other procedures for testing the hypothesis about the number of indigenous errors are desirable.

MILLS discusses a simple procedure for testing the hypothesis that the number of indigenous errors is less than or equal to k . We outline his procedure, called the Assert, Insert and Test (AIT) process .

(1) Assert that $n_i \leq k$.

(2) Insert n_s seeded errors.

(3) Test until all n_s seeded errors are found and record the number of indigenous errors found, say i .

(4) Compute $C(n_s,k)$, the confidence with which the assertion $n_i \leq k$ is rejected, as

$$C(n_s,k) = \begin{cases} 1 & \text{if } i > k \\ \dfrac{n_s}{n_s + k + 1} & \text{if } i \leq k \end{cases} \qquad (4.4.4)$$

The confidence $C(n_s,k)$ is the probability that an AIT process will correctly reject a false assertion and is conservative in the sense that $C(n_s,k)$ is the

power of the test evaluated at $n_i = k + 1$. On observing (4.4.4) it is obvious that our confidence increases with larger values of n_s and decreases with increasing values of k . This is illustrated in Table 1 .

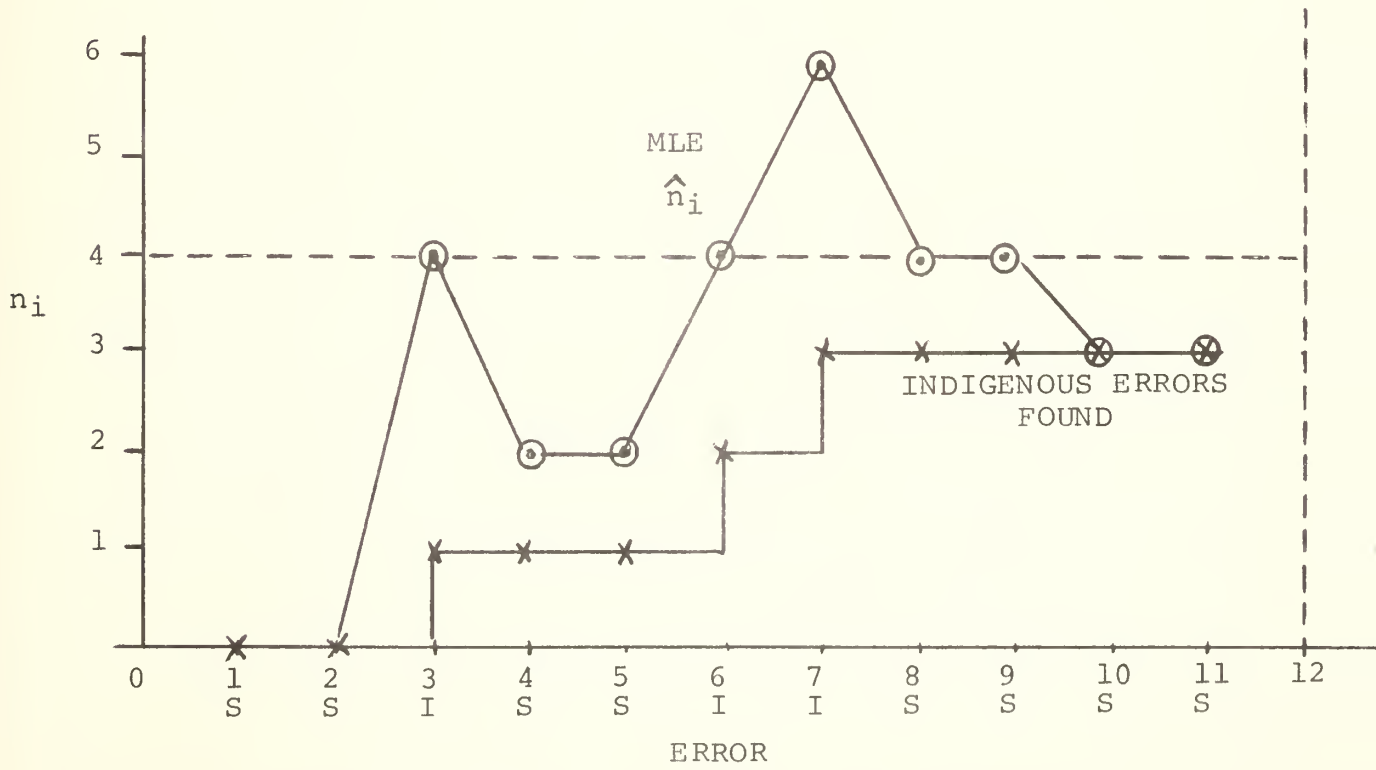
n_s \ k	0	1	2	3	4
1	.50	.33	.25	.20	.17
2	.67	.50	.40	.33	.29
3	.75	.60	.50	.43	.38
4	.80	.67	.57	.50	.46
5	.83	.71	.62	.56	.50
10	.91	.83	.77	.71	.67

AIT Confidence $(i \leq k)$

Table 1

MILLS suggests that an AIT chart be maintained to provide a visual progress report. The chart gives a chronological record of both the maximum likelihood estimate of the number of indigenous errors and the actual number of indigenous errors found. The example below illustrates an AIT chart.

Suppose that the null hypothesis (assertion) is H_0: $n_i = 4$ and 8 errors are seeded Then, if the number of indigenous errors found is greater than 4 (the total number of errors found is greater than $n_s + k = 12$), H_0 can be rejected with certainty. Let us now suppose that the AIT process produces the fullowing sequence of errors: S, S, I, S, S, I, I, S, S, S, S where S represents a seeded error and I an indigenous error. The AIT chart for this test is shown below.



AIT Chart (n_S= 8, k = 4, Confidence = 0.62)

Figure 9

The test succeeded since the MLE curve ended up beneath the horizontal line $n_i = 4$. After a while, the MLE curve should appear as random fluctuations about the horizontal line at height n_i with decreasing variance.

MILLS' AIT process can be extended easily to allow for different stopping rules other than waiting until all seeded errors are found. One useful modification is to stop after a fixed number j of the seeded errors $(j < n_s)$ have been found. If this is the case, the maximum likelihood estimate is unaffected, but the confidence is now.

$$C'(n_s,k,j) = \begin{cases} 1 & \text{if } i > k \\ \dfrac{\binom{n_s}{j-1}}{\binom{n_s + k + 1}{k+j}} & \text{if } i \leq k \end{cases}$$

Other interesting modifications might have the AIT process stop after a fixed number of errors of either type have been found, or to stop after a fixed number of indigenous errors have been found. For each modification a new confidence equation must be determined. Other error-seeding models with different underlying assumptions have been investigated by LIPOW [13] .

The error-seeding models are intuitively appealing and they have the advantage of being quite simple computationally. Nevertheless, there are some problems involved with the insertion of errors. The models assume that, at each run, all remaining errors are equally likely to be found. Thus, the errors must be inserted in such a way that the testing process

is not biased toward either the seeded errors or the indigenous errors. This is a nontrivial problem in itself because the nature of the indigenous errors is unknown. If substantive error data existed the seeded errors could be set to reflect actual experience. Much research must be done before a methodology for introducing software errors satisfying the assumptions of the error-seeding model can be developed. If that problem can be solved, the error-seeding program offers a powerful tool for validating computer programs.

4.5 A Simple Reliability Model for Qualitative Data

Up to this point we have assumed that data are in the form of times between failures. Occasionally the data is not of that type, or the time-to-failure data (quantitative data) has been used solely for the purpose of classifying a test run as a success or a failure. Such a classification may be necessary because the form of the mathematical model which describes times-to-failure is unknown or intractable. Another reason may be that it is costly or simply not feasible to install the recording equipment or hire observers to monitor the software continuously as needed to obtain the variable data. Thus, we may have to be satisfied with counting the number of failures in a given number of test runs or the number of failures during test intervals of a given length of time.

If we are reasonably confident that the quantitative data follow a known, simple, mathematical form, we should use the quantitative data for estimating reliability. The quantitative data allow for more precise observation and make more efficient use of the information available. This efficient use of data is of increasing importance as experimentation becomes more and more costly. Nevertheless, if the quantitative data is not available, we must make do with what we can get. Reliability can still be demonstrated when

qualitative data is used. Suppose, for example, that the required failure-free time of operation is T and several runs are made for T units of time noting only whether each run was a success or a failure. Then, the simple binomial distribution can be used to estimate the reliability. Other reliability models depending on qualitative data will be described in this section.

We allow some flexibility in the classification of a run as a success or a failure. A success may mean zero software failures of any type; it may mean no software failures of a given criticality or worse; it may mean that the total number of failures is not greater than some given number; or it may be taken to mean whatever the user desires. We assume that the user has settled on a definition of success and on a definition of "test run".

We now describe some reliability models which make use of qualitative data. They are somewhat heuristic models which, in some cases, require a subjective assessment of the test runs. First, we describe a very general model proposed by Mac WILLIAMS [17] . Then we look at some special cases.

Suppose that N test runs are conducted and let n_i be the number of failures observed in test i . Let $E_i(n_i)$ be a measure of the performance during the i^{th} test and let $W_i(n_i)$ be a weighting factor which reflects seriousness of the errors observed in the i^{th} test. We require that

$$0 \leq E_i(n_i) \leq 1 , \quad E_i(0) = 1$$

and (4.5.1)

$$0 \leq W_i(n_i) \leq 1 , \quad W_i(0) = 1$$

and we take

$$R = \frac{1}{N} \sum_{i=1}^{N} E_i(n_i) \cdot W_i(n_i) \qquad (4.5.2)$$

to be an estimate of the reliability.

If we take $E_i(n) = 0$ for all $n > 0$ and $W_i(n) = 1$ for $n \geq 0$, we obtain the special case where reliability is estimated to be the fraction of successful runs. For example, if in 100 test runs 85 were successful, we would estimate the reliability to be $R = 0.85$.

As another example, suppose that failures have been classified according to severity as low, medium and high. Let $E_i(n_i) = 1$ for all n_i, and let the weights given to low, medium and high severity errors be 0.9, 0.1 and 0 respectively. Define $W_i(n_i)$ to be the product of the weights assigned to the n_i errors and $W_i(0) = 1$. For example, if test i results in 2 low severity errors and 1 medium severity error, then $n_i = 3$ and $W_i(n_i) = (0.9)(0.9)(0.1) = .081$. In this case the reliability model (4.5.2) lets the user weight errors according to their impact on the performance of the software.

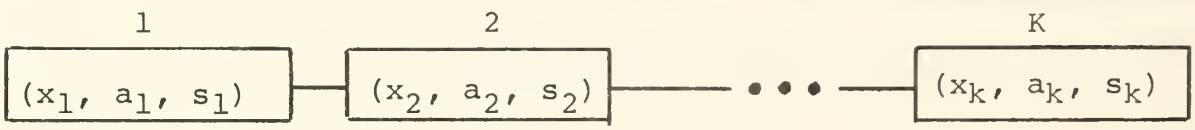
The flexibility of (4.5.2) makes it particularly attractive. Test personnel are not "locked" into complete objectivity. Instead, they are allowed to interject their subjective assessment of software performance. This allows personnel to adapt the model to their particular needs. The model (4.5.2) could even be used when quantitative data are available if

it were desirable to weight subjectively the quality of each test run in a series of runs.

4.6 A Reliability Growth Model for Qualitative Data

The reliability growth model (4.3.4) which utilized times-to-failure data allowed us to account for the natural growth of reliability that takes place as errors are detected and corrected. We now discuss a model which relies on qualitative data and which explicitly accounts for the reliability growth. The model is one developed for hardware systems by BARLOW and SCHEUER [1], but it appears to be adaptable to software.

We consider a trial to be a test run of length T . Imagine a testing program which consists of K stages with n_i trials at the i^{th} stage. Both K and the n_i's are completely arbitrary, therefore, no control over the length of a sampling interval is required. Each trial is considered to be a success or a failure. At the end of each of the k stages, an effort is made to determine the cause of each failure and to correct the software so that the failure will not reoccur. We allow for the possibility that the causes of some failures may escape our detection. Consequently, those failures might reappear. Using the terminology of BARLOW and SCHEUER, we classify failures as "assignable-cause" or "inherent" failures depending on whether the cause is determined and removed or not. For each stage, we record the number of inherent failures X_i, the number of assignable-cause failures A_i , and the number of successful trials S_i as illustrated in Figure 10.



FAILURE RECORD DURING K-STAGE PROGRAM

Figure 10

Define q_o to be the probability of an inherent failure during a trial (a run of length T). Since no corrections are made of these failures, q_o is assumed constant throughout the K-stage test program. On the other hand, the probability of assignable-cause failures should decrease from one stage to the next since the causes are assumed to be removed. Let q_i be the probability of an assignable-cause failure in the i^{th} stage. We assume that $q_i \leq q_{i-1}$ for $i = 2, \ldots, K$. The probability of success for a trial, the reliability for time T, is

$$R_i = 1 - q_o - q_i .$$

The probability of X_i inherent failures, A_i assignable-cause failures and S_i successes in $X_i + A_i + S_i$ total trials during the i^{th} stage is given by the multinomial probability function

$$\frac{(X_i + A_i + S_i)!}{X_i!\, A_i!\, S_i!} q_o^{X_i} q_i^{A_i} (1 - q_o - q_i)^{S_i} .$$

The likelihood function for the K stages is

$$L(X_1, A_1, S_1; \ldots; X_K, A_K, S_K)$$

$$= \prod_{i=1}^{K} \frac{(X_i + A_i + S_i)!}{X_i!\, A_i!\, S_i!}\, q_0^{X_i} q_i^{A_i} (1 - q_0 - q_i)^{S_i} \qquad (4.6.1)$$

From (4.6.1) the maximum likelihood estimators of q_0 and q_i are easily shown to be

$$\hat{q}_0 = \sum_{i=1}^{K} X_i \Big/ \sum_{i=1}^{K} (X_i + A_i + S_i) \qquad (4.6.2)$$

and

$$\hat{q}_i = (1 - \hat{q}_0) A_i / (A_i + S_i) \qquad (4.6.3)$$

for $i = 1, 2, \ldots, K$.

Instead of the estimates $\hat{q}_i$ we want the maximum likelihood estimates of the q_i's subject to the restriction that $q_1 \geq q_2 \geq \ldots \geq q_K$. Let $\bar{q}_i$ be the MLE of q_i subject to this condition. Then, BARLOW and SCHEUER show that

$$\bar{q}_i = (1 - \hat{q}_0) \max_{u \geq i} \min_{r \leq i} \frac{A_r + \ldots + A_u}{(A_r + S_r) + \ldots + (A_u + S_u)} \qquad (4.6.4)$$

Equation (4.6.4) gives an explicit expression for $\bar{q}_i$. However, in practice one would probably want to determine the $\bar{q}_i$'s using the following equivalent procedure. If $\hat{q}_1 \geq \hat{q}_2 \geq \ldots \geq \hat{q}_K$ then $\bar{q}_i = \hat{q}_i$ for $i = 1, 2, \ldots, K$. If $\hat{q}_j < \hat{q}_{j+1}$ for some j , then combine the observations in the j^{th} and $(j + 1)^{st}$ stages and compute the MLE of the q_i's by (4.6.3) for the $K - 1$ stages thus formed. Continue this procedure until the estimates of the q_i's form a non-increasing sequence. These estimates are the maximum likelihood estimates of the q_i's subject to $q_1 \geq q_2 \geq \ldots \geq q_K$.

Example: A software test program consisting of six stages was conducted. Each stage was terminated when the number of assignable-cause failures reached three. At the conclusion of each stage, a debug effort eliminated the source of all assignable-cause failures so that the software was different in each succeeding stage, but homogeneous within any given stage. Table 2 summarizes the results at the six stages.

STAGE	INHERENT	ASSIGNABLE CAUSE	SUCCESSES	TRIALS	$\frac{a_i}{a_i+s_i}$
i	x_i	a_i	s_i	n_i	
1	0	3	1	4	3/4
2	1	3	3	7	1/2
3	1	3	8	11	3/11]6/19
4	0	3	5	9	3/8
5	0	3	11	14	3/14
6	2	3	30	35	3/33
TOTALS	4	18	48	80	

SIX STAGE TEST RESULTS

Table 2

The MLE of q_o does not depend on the groupings of the data into stages and is given by:

$$\hat{q}_o = \sum_{i=1}^{6} X_i / \sum_{i=1}^{6} n_i = 4/80 = .05$$

To determine the MLE of the q_i's subject to the restriction that they be non-increasing we aggregate those stages where the non-increasingness is violated until a non-increasing sequence is obtained. Since the estimate of q_o is not stage dependent, it suffices to look at the ratios $A_i/(A_i + S_i)$.

Observe from Table 2 that there is a reversal in non-increasingness from stage three to stage four. That is, $\hat{q}_4 > \hat{q}_3$. Therefore we combine stages three and four and get a new estimate

$$A_3 + A_4/[(A_3 + S_3) + (A_4 + S_4)] = 6/19$$

We now have the new sequence of ratios:

$$3/4,\ 1/2,\ 6/19,\ 3/14,\ 3/33$$

which has the required non-increasing property. Therefore the MLE's of

the q_i's , subject to the non-increasing condition, are

$$\bar{q}_1 = (.95)(3/4) = .713 \qquad \bar{q}_4 = (.95)(6/19) = .300$$

$$\bar{q}_2 = (.95)(1/2) = .475 \qquad \bar{q}_5 = (.95)(3/14) = .202$$

$$\bar{q}_3 = (.95)(6/19) = .300 \qquad \bar{q}_6 = (.95)(3/33) = .086$$

The MLE of the reliability of the software in its last test stage is

$$\bar{R}_6 = 1 - \bar{q}_0 - \bar{q}_6 = 0.864 \ .$$

If all test stages were incorrectly assumed to be homogeneous and all data pooled together, the estimate of reliability, making no distinction between assignable-cause failure and inherent failure, would be

$$R'_6 = \sum_{i=1}^{6} S_i / \sum_{i=1}^{6} n_i = 48/80 = 0.60 \ .$$

We see that consideration of the reliability growth has a substantial impact on the estimate of reliability.

BARLOW and SCHEUER also derive a lower confidence bound for software reliability. They show that a $100(1 - \alpha)$ percent lower confidence bound on R_K, the software reliability in its final configuration, having observed S_i successes in n_i trials at stage i, is found by setting

$$S = \sum_{i=1}^{K} S_i \quad \text{and} \quad n = \sum_{i=1}^{K} n_i$$

and determining R_*, the largest R such that

$$\sum_{j=0}^{S-1} \binom{n}{j} R^j (1-R)^{n-j} \geq 1 - \alpha . \qquad (4.6.6)$$

The value R_* is easily found using binomial tables. R_* is the desired confidence bound in the sense that

$$P[R_K \geq R_* \mid R_1 \leq R_2 \leq \ldots \leq R_k] \geq 1 - \alpha .$$

BARLOW and SCHEUER also show that the bound is the best that can be achieved under our assumptions of non-decreasing values of the R_i's. In the above

example, the 95% lower confidence interval for R_6 is given by

$$R_* = 0.803$$

In this reliability growth model we have allowed for the possibility that some errors may go uncorrected. There is, of course, no requirement that any "inherent" failures occur. Indeed, it is to be expected that most causes can be determined and the source of the troubles removed. Most likely, if any inherent errors do exist, they are those rare objects which are not observed until most assignable-cause errors have been removed.

4.7 Bayesian Reliability Models

We complete our treatment of software reliability models with a brief discussion of Bayesian reliability models. The Bayesian approach has been widely used to develop reliability models for hardware systems. The approach would also seem to be applicable for software reliability.

Suppose we have collected the data $t_1, t_2, \ldots, t_n$ representing the times between failures for the first n observed software failures. We want to make an inference about the distribution of the time to the $(n + 1)^{th}$ failure. Let the random variable T_i be the run time between the $(i - 1)^{th}$ and the i^{th} failures and let T_i and T_j be independent for all $i \neq j$. Let $f(t|\lambda(i))$ be the probability density function for T_i with parameter $\lambda(i)$. We write the parameter as a function of i to allow for changes that may occur as errors are removed. If one supposes that $\lambda(i)$ is a

failure-rate parameter and the software is debugged after each failure occurs, then one would expect that $\lambda(i) \geq \lambda(i+1)$ (the failure rate is non-increasing). Let $g(\ell|i,\beta)$ be the probability density function for the random variable $\lambda(i)$, where β is a parameter (or vector of parameters). The Bayesian approach treats the parameter β as though it were a random variable. The analyst begins with a "prior" distribution for β, expressed by the density function $h_o(\beta)$ and then uses the failure data to update the prior and obtain a "posterior" distribution $h_1(\beta)$. Using the Bayes' Theorem, the relationship between the prior and the posterior is given by

$$h_1(\beta) = \frac{\prod_{i=1}^{n} \int f(t_i|\lambda) g(\lambda|i,\beta) d\lambda \cdot h_o(\beta)}{\int [\prod_{i=1}^{n} \int f(t_i|\lambda) g(\lambda|i,\beta) d\lambda \cdot h_o(\beta)] d\beta}$$

Having obtained the posterior probability function $h_1(\beta)$, the probability density function of $\lambda(n+1)$ and T_{n+1} are then found to be

$$g(\ell|n+1) = \int g(\ell|n+1,\beta) h_1(\beta) d\beta$$

and

$$f(t_{n+1}|\lambda(n+1)) = \int f(t_{n+1}|\ell)g(\ell|n+1)d\ell$$

The integrals above (all taken to range over the interval $(-\infty, \infty)$) appear to be rather awesome mathematically. Indeed, the mathematical difficulties and the problem of choosing an appropriate prior distribution for β have impeded the acceptance of the Bayesian approach. Nevertheless, for some special cases of practical interest, the mathematics works out quite nicely. For example, LITTLEWOOD and VERRALL [14] obtain tractable results for the case

$$f(t|\lambda) = \begin{cases} \lambda e^{-\lambda t} & t > 0 \\ 0 & t \leq 0 \end{cases}$$

$$g(\ell|i,\beta) = \begin{cases} \dfrac{\psi(i)[\psi(i)\ell]^{\beta-1}e^{-\psi(i)\ell}}{\Gamma(\beta)} & \ell > 0 \\ 0 & \ell = 0 \end{cases}$$

Where $\psi(i)$ is assumed to be a known monotonically increasing function. (Software repairs are undertaken after each observed failure). Assuming a uniform prior distribution for β, they derive the following:

$$F(t_{n+1}) = P[T_{n+1} \leq t_{n+1}]$$

$$= 1 - \left(\frac{\alpha}{\alpha + \ln(1/k_{n+1})}\right)^{n+1}$$

where $k_i = \psi(i)/(\psi(i) + t_i)$, $i = 1, 2, \ldots, n + 1$ and

$$\alpha = \ln \left(\prod_{i=1}^{n} k_i \right) .$$

With the distribution $F(t_{n+1})$ in hand, the reliability function is simply $R(t_{n+1}) = 1 - F(t_{n+1})$.

THOMPSON and WALSH [29] also have applied the Bayesian approach to the software reliability problem. Instead of looking at the failure rate or the time-to-failure distribution, they apply the approach directly to the reliability function. That is, they treat the true but unknown reliability function, R , as a random variable whose probability function is obtained from Bayes Theorem using test data. Let $P(R)$ be the prior density of the reliability which has been obtained subjectively. Also, let $\hat{R}$ be the estimate of the reliability obtained from the test data and let $g(\hat{R}|R)$ be the conditional density of the estimate $\hat{R}$ given that the reliability is R . Then, from Bayes Theorem, the posterior density of the reliability R is

$$f(R|\hat{R}) = \frac{g(\hat{R}|R)p(R)}{\int_0^1 g(\hat{R}|R)p(R)dR} \qquad (4.7.1)$$

Integrating (4.7.1), we get the distribution function

$$F(R) = \int_0^R f(X|\hat{R})dX \qquad (4.7.2)$$

From (4.7.2) Bayesian confidence limits on reliability can then be obtained. For example, the lower $100(1 - \alpha)\%$ confidence limit is that value $R_{1-\alpha}$ such that $F(R_{1-\alpha}) = 1 - \alpha$. THOMPSON and WALSH present two special cases - one using qualitative data and the other quantitative data. In the first case, they assume that the probability of a software failure over a given test interval is constant and that in n runs s were successes. Taking $\hat{R} = s/n$ and a uniform prior distribution $p(R)$, they show that the posterior density of R is the beta density

$$f(R|\hat{R}) = \frac{1}{\beta(s+1,\ n-(s+1))}\ R^s(1-R)^{n-s}$$

where

$$\beta(s + 1,\ n - (s + 1)) = \frac{(n + 1)!}{s!(n - s)!}$$

In the second case they assumed a constant failure rate and used as their estimate of reliability

$$\hat{R}(t) = \exp(-\, rt/\hat{T})$$

where $\hat{T}$ is the total run time and r is the number of failures. Again using a uniform prior, they show that

$$f(R(t) \mid \hat{R}(t)) = \frac{(1 + \hat{T}/t)^{1+r}}{\Gamma(1+r)} \; R^{\hat{T}/t} \; (\ln 1/R)^{r}$$

At present, the Bayesian approach seems to suffer in comparison to the simplicity of the other approaches we have presented. In addition, it requires the subjective assessment of the analyst in determining the prior distributions. The Bayesian models require more structure than do the other models. For these reasons, the Bayesian models do not appear to be as useful as some of the others we have considered.

V. GUIDELINES FOR SOFTWARE QUALITY ASSURANCE

5.1 Introduction

We have discussed testing, and several methods for estimating software reliability have been presented. For the most part, these methods are most useful for the operational tests which are conducted by the user or some independent test agency after the integration stage of software development. However, they are also appropriate for use at the earlier stages where it may be desirable to estimate module or process reliability or to give guidelines on the amount of additional testing that should be given a certain portion of the software. Nevertheless, it is usually the case in actual practice that the software delivered to the customer is full of bugs. This is true even though the software developer may have been required to demonstrate through some sort of formal qualifying test that his product performs as specified.

The delivery of unreliable software to the customer results in his loss of confidence in the product. The customer finds himself forced to conduct extensive testing to increase its reliability, or to put the unreliable software into use and correct the errors in the time-honored fashion of fixing them as they appear. Neither of these alternatives is satisfactory to the user. He feels that the developer is better equipped to correct software errors and that it is the developers job to release a reliable product. It is not unreasonable for the user to feel this way.

What is needed is that more effort be funneled in the direction of ensuring that proper management techniques be performed early in the development of the software so that the user is not forced to accept less than

what he bought. Since part of the problem results from poor programming practices, research is also needed in the development of tools which enable a programmer to write better software.

5.2 Design for Reliability

One way to improve the quality of software is to expend more effort and resources in its design. If causes of software unreliability can be determined, then steps can be taken to alleviate the contributions of those causes. Testing and debugging alone cannot ever guarantee completely reliable software for hidden errors can sometimes violate the system without ever giving a warning (no failures are observed). DIJKSTRA [4] has remarked, "Testing can be used to show the presence of bugs but never to show their absence." What, then, can be done in the design area that will reduce the number of errors written into the software?

TSICHRITZIS and BALLARD [31] suggest that the software can be structured in such a way as to enhance its reliability. First, a study of the relative frequencies of different types of errors in programs could identify the most frequent characteristic errors. This information would then, in turn, be used to determine the program structures and languages which are most reliable. Those structures which are unreliable should be avoided, when possible, by programmers. Next, the software could be structured so as to allow easier and more complete testing. This could be accomplished by building up the software from modules and using protection mechanisms which establish boundaries and rules to restrict communication between the various modules. The object of the protection mechanism is to minimize

interface problems. By controlling the interactions between the various parts of the software, the protection system would tend to isolate errors so that an error which occurs in one part cannot damage the other parts. The system would create "fire walls" which would facilitate the recovery from failures. In addition, the protection system would aid in the discovery and location of errors. This would happen because of the failures that result from an attempt to violate some protection mechanism.

Software testing at the lowest level of the software hierarchy can often be nearly exhaustive in the sense that all logic paths of control can (and should) be checked at least once. Testing the individual paths with sample inputs covering the extreme values in the domain of the variables will provide a good start on assuring good quality. Historically, the trouble with modular test approach has been that problems result when the modules (processes) are integrated so that they must interact with other modules (processes). However, if a good protection system could be developed the interaction problems would be minimized and modular testing would be feasible. Then, the software could be designed and structured so that the modules can be verified independently of the higher levels and then used without further testing to verify the next level. Because of the reduction in the number of tests that would be required, huge savings in both time and dollars could be achieved if validation could be accomplished by testing modules independently. For example, suppose module i has n paths to test and module i is called from m places in module j. If the modules can be tested independently, we need only test n paths in module i and m paths in module j, a total of $m + n$ paths. However, if they

must be combined for testing, n x m paths must be checked. When one considers that n and m may range upwards into the hundreds or thousands, the potential savings are obvious.

Since its inception, programming has been considered an art rather than a science. Much work is needed to develop a set of conceptual and operational principles that constitute good programming practice so that programming can be placed under tighter control. Considering the current high cost of reliability, concentration in this area may offer the highest ratio of benefits to cost.

5.3 The User's Role in Software Development

Unless programming becomes more of a science, the user will have little control over actual programming practice. However, there is much that the user can do to improve the quality of the software he receives. As more and more large software subsystems are developed and lessons learned from the successes and failures of the development programs, practices which constitute good software control are evolving. Software acquirers are recognizing the value of their practical experience with software development and have begun to share their experiences, both good and bad, in the open literature (see, for example, BUCKLEY [2] , KEEZER [11], COUTINHO [3], or ELLINGSON [5]). In the time-honored fashion of trial and error many guidelines have developed.

Official guidelines for government agencies involved as acquirers in the software development process are given by two documents, MIL-STD-490 (Specification Practices) and MIL-STD-483 (Configuration Management Practices of Systems, Equipment, Munitions and Computer Programs). The Specification

Practices document establishes the detailed format and content of specifications for computer programs. It includes both a Computer Program Development Specification and a Product Specification. The development specification describes the performance requirements necessary to design and verify the computer program in terms of performance criteria. The product specification is the document representation of the computer program; it consists of the flow charts and narrative that logically describe the computer program, the coding and the data. The Configuration Management standard expands on the documentation requirements and provides uniform procedures for preparing, formating and processing changes to computer programs once a configuration is fixed.

Although certainly deserving of much attention, we shall not attempt to discuss extensively the steps a software acquirer can and should take to assure receipt of a quality product. Such an important subject is better discussed by people who have practical experience such as those cited above. Nevertheless, we shall focus on a few areas which fall under the responsibility of the software acquirer and which have a potentially large impact on the quality of the software.

Many of the problems that result can be traced to the very outset of the software development program. Too often the customer, who is responsible for establishing detailed performance specifications, will write vague statements as to what he wants. This forces the contractor to guess at what is wanted or allows him too much freedom for personal interpretation of what is desired. The performance specifications should be much more than mere expressions of good intentions. They must describe exactly

those functions that must be performed, and they must contain enough information to enable a contractor to transform an operational need or system requirement into a design specification. At the same time, overall software test plans must be written at the same level of detail. These form the basis for the development of future test plans and procedures. The test plans should require that each performance requirement of each computer program configuration end-item be verified in some appropriate manner, and they should specify the acceptance criteria. Detailed and complete test plans not only give the contractor something to shoot for, but they also provide the contractor's personnel with specific tests against which they can check their work as they progress with the coding.

In order to improve visibility, the customer must require configuration management control and detailed documentation. This allows the customer to monitor the progress in the development of the product and to identify and control changes to be made to an already approved specification. Further, it enables management to see what they are managing. The documentation requirement is necessary because it fosters better communication, and documentation is itself an important part of the software end product.

We have already discussed some of the problems experienced by test personnel during operational testing caused by the lack of an overall accepted methodology of software testing. The same problems thwart the success of the verification testing conducted by the contractor. Traditionally, contractors have attempted to verify that the software performs according to specifications by demonstrating its performance using "canned" tests with exactly-prescribed inputs. The problem is that the software passes the formal

verification tests, but it often will fail to process properly other inputs under varying load conditions. Consequently, when the entire system undergoes operational testing numerous problems surface. A partial solution to this problem reverts to the proper writing of performance specifications and test plans and procedures. Much thought needs to be given to the acceptance criteria that are written into the software contract. The customer must state explicitly how the contractor must verify each performance specification.

Because software is a much less tangible product than is hardware, the customer must obtain visibility into the contractor's efforts from the very beginning of the contract. Although there is little that can be observed or inspected in order to assess intermediate progress, the customer should strive to actively participate in the development by furnishing guidance to the contractor during design reviews and software testing. The customer should obtain sufficient visibility to enable him to ensure that the contractor establishes and enforces good management control and procedures.

Lastly, even with active participation by the customer and the best management, the delivered software will not be perfect. Therefore, the customer must expect to require some maintenance after he accepts the product, and he should provide for the contractor to be kept on the job for some period after acceptance to correct problems as they occur.

VI. CONCLUSIONS

In this paper we have looked at some of the problems experienced in the computer software development process. We have pointed out the importance of software quality assurance and the necessity for keeping close tabs on the software quality. Using the natural analogies between software and hardware, we have defined a measure of effectiveness. Test procedures and data collection requirements have been described.

Several mathematical models which convert the raw software failure data into estimates of software reliability have been presented. The mathematical models cover different types of failure data and, in some cases, yield not only point estimates of reliability, but also estimates of the number of errors remaining in the software subsystem (or a part of that subsystem) and confidence intervals for reliability. The models represent a survey of the state of the art in reliability estimation. With only a few exceptions, they have not been validated with actual software failure data. They are presented here because they appear to the author to be mathematically plausible and intuitively appealing. Nevertheless, they have not yet passed the "acid test" -- good performance in actual use. Consequently, much work remains to be done before any particular one can be recommended strongly. We do recommend that software data of the form described in this report be collected and the various models be tested against real data.

In addition to the need for a validation effort on the reliability models, we have identified other areas related to the software problem which cry out for further work. For example, work needs to be done to identify

unreliable program structures and to develop a set of operational principles that constitute good programming practice, so that better software will be written. Also, there is a need for the development of a methodology for software testing. This is critical when one considers the huge portion of total software development costs which are consumed by the test effort.

Finally, we have pointed out some "management" responsibilities of the software customer which can have a significant impact on the quality of the product he is delivered. At the outset he is responsible for the writing of contractual performance specifications, test plans and procedures, and acceptance criteria. Much work needs to be done in this area to head off those problems which result because specifications, plans and requirements contain too little detail. After the customer has written down exactly what he wants and requires, his function is primarily one of vigilance.

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APPENDIX

A brief summary of the more important terms associated with mathematical reliability theory is presented in the following for those readers unfamiliar with that theory.

Reliability.

The reliability of a product is defined as the probability that the product will function within specified limits for at least a specified period of time under specified environmental conditions.

Various probability distributions are required for the study of reliability. Of fundamental importance is the probability distribution of the time to failure.

Time-to-Failure Distribution.

Let $f(t)$ be the probability density of the time to failure of the product, that is the probability that the product will fail between times t and $t + \Delta t$ is given by $f(t) \cdot \Delta t$. The probability that the product will fail sometime before time t is given by

$$F(t) = \int_0^t f(s)ds$$

which is sometimes called the "unreliability" function.

Reliability Function.

The probability that the product will survive to time t is given by

the reliability function

$$R(t) = 1 - F(t)$$

Note the relationships between $f(t)$, $F(t)$ and $R(t)$. In particular

$$f(t) = dF(t)/dt = - dR(t)/dt$$

Instantaneous Failure Rate.

Another probability function which is convenient for use in reliability studies is the conditional probability that the product will fail in the interval $(t,t+\Delta t]$ given that the product has survived to time t. Mathematically, this probability is given by

$$(F(t+\Delta t) - F(t))/R(t)$$

On dividing by the length of the interval, Δt, and taking the limit as Δt goes to zero we get the instantaneous failure rate or hazard rate

$$Z(t) = \lim_{\Delta t \to 0} \frac{F(t+\Delta t) - F(t)}{\Delta t} \cdot \frac{1}{R(t)} = \frac{dF(t)/dt}{R(t)}$$

Using the identities involving $f(t)$, $F(t)$ and $R(t)$ we get the following equivalent expressions for $Z(t)$:

$$Z(t) = f(t)/R(t)$$

$$= \frac{- dR(t)/dt}{R(t)}$$

$$= - \frac{d}{dt} \ln[R(t)]$$

This differential equation can be solved for $R(t)$ to yield

$$R(t) = \exp(-\int_0^t Z(s)ds) \ ,$$

and since $Z(t) = f(t)/R(t)$ we get

$$f(t) = Z(t) \exp(-\int_0^t Z(s)ds) \qquad (A1)$$

Expression A1 shows how the time to failure density is related to the instantaneous failure rate function.

Mean Time to Failure.

A measure of effectiveness often required in reliability studies is the mean time to failure (MTTF). This is easily found by taking the first moment of the time to failure distribution. In terms of the density $f(t)$,

$$MTTF = \int_0^\infty tf(t)dt$$

An equivalent expression giving the MTTF in terms of the reliability function is

$$MTTF = \int_0^\infty R(t)dt \ .$$

The Exponential Model.

In many reliability studies the product reaches a point in its life cycle where the failure rate is constant, that is,

$$Z(t) = \alpha \ , \qquad \alpha > 0$$

On substituting into equation A1 we get the time-to-failure density

$$f(t) = \alpha \exp(-\alpha t) \qquad t > 0$$

which is the well known exponential probability density. Simple calculations show that

$$R(t) = \exp(-\alpha t)$$

and

$$MTTF = 1/\alpha$$

An important property of the exponential model is its "loss of memory." In words, this means that the probability that the product will survive an additional t_0 units of time does not depend on the amount of time the product has already been in operation. Mathematically, if T is the time-to-failure random variable, then

$$P[T > t_0 + s \mid T > s] = P[T > t_0] = R(t_0) = \exp(-\alpha t_0) .$$

The exponential model is easily the most widely applied model in reliability analyses. There are several reasons for its acceptability. First, its theoretical properties such as constant failure rate and loss of memory often seem appropriate for describing the probabilistic characteristics of the failures of some products. Indeed, many theoretical investigations and experimental verifications have shown that, for some products, the time-to-failure density is $f(t) = \alpha \exp(-\alpha t)$. In the second place, the popularity of the exponential model is often due to its simplicity. It has only a single parameter, and many problems can be solved in explicit form using

simple equations.

Recently, the exponential model has been the subject of criticism because it is being accepted uncritically. Analysts must attempt to justify its use on the basis of the nature of the failures of the product under consideration. In many cases failures are described quite well by the exponential model, but there are also many cases where it is not appropriate.

The Weibull Model.

While the assumption of a constant failure rate is often appropriate for describing chance failures, it is not always sufficient. This particularly true during the early "burn-in" stage and the late "wear-out" stage. Nor would the constant failure rate be appropriate during a period of reliability growth due to improvements in the product. Thus, other mathematical functions are required so that increasing or decreasing failure rates can be considered. A versatile function often used to approximate such failure rates is

$$Z(t) = \alpha\beta t^{\beta-1} \; ; \qquad t > 0 \; ; \quad \alpha,\beta > 0 \; .$$

When $\beta < 1$ the failure rate decreases with time; if $\beta > 1$ it increases with time; and if $\beta = 1$ the failure rate is constant. Solving for the time-to-failure density gives

$$f(t) = \alpha\beta t^{\beta-1} \exp(-\alpha t^{\beta}) \qquad t > 0$$

This density is called the Weibull density. The reliability function is

given by

$$R(t) = \exp(-\alpha t^{\beta}) \ .$$

Like the exponential model, the Weibull enjoys widespread use. It can also be justified on theoretical grounds, for it has been shown that the limiting distribution (as $n \to \infty$) of the time to failure of the minimum of n independent random variables each having the same distribution is described by the Weibull law. Perhaps more than any other reason, the Weibull model has been used because it is so versatile. The Weibull faimly is so rich that some member of the family can usually be found to describe the nature of failures for most products.

Other Models.

Other probability functions which have been found useful for describing the random nature of failures are the gamma and the lognormal. These functions are represented by:

$$\text{Gamma:} \quad f(t) = \frac{t^{\alpha-1} \exp(-t/\beta)}{\Gamma(\alpha)\beta^{\alpha}} \quad ; \quad \alpha\beta > 0$$

and

$$\text{Lognormal:} \quad f(t) = \frac{1}{\sqrt{2\pi}\ \beta t} \exp(-\ln\ (t-\alpha)^2/2\beta^2); \ \beta > 0$$

For appropriate choices of parameters, these functions can be made to represent either increasing or decreasing failure rates.

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